SEMINAR REPORT ON

***Granger Causality Test***

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**Presented by :** ***RAVAL SUJIT***

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**INTRODUCTION**

* Granger causality is a statistical concept of causality that is based on prediction.
* Prof. Clive W.J. Granger, recipient of the 2003 Nobel Prize in Economics developed the concept of causality to improve the performance of forecasting.
* The Granger causality test is a statistical hypothesis test for determining whether one time series is useful in forecasting another.
* In time series analysis, sometimes, we would like to know whether changes in a variable will have an impact on changes other variables.
* To find out this phenomena more accurately, we need to learn more about Granger Causality Test (Granger, 1969).

## ****Overview of Time series:****

**Time series is a set of observations on the values that the variable takes at different times. The time interval at which the data is collected may be hourly, daily, weekly, monthly, quarterly, annually, etc.**

**Time series data can be categorized as follows:**

* **Univariate time series – Consists of a single variable that changes over time. Eg Account balance over a span of ten years.**
* **Multivariate time series – Consists of more than one variable which varies over time and there may be dependency among the variables. Eg Different expenditures of a family over a span of ten years.**

Time series analysis provides insight into the pattern or characteristics of time series data. Time series data can be decomposed into three components:

* Trend – This shows the tendency of the data over a long period of time, it may be upward, downward, or stable.
* Seasonality – It is the variation that occurs in a periodic manner and repeats each year.
* Noise or Random – Fluctuations in the data that are erratic.

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**What is Causality ?**

Causality is the relation between cause and effect. It refers to the relationship between events Where one set of events(the effects) is a direct consequence of another set of events(the causes).Causal inference is the process by which one can use data to make claims about causal relationships.

**Types of Causality**

(1).**Uni Directional Causality** : GDP🡪Stock prize

->GDP cause Stock prize but stock prize do not cause GDP then called unidirectional causality.

(2).**Bi-Directional Causality :**GDP🡨🡪Stock prize

-> GDP cause stock prize and Stock prize cause GDP then called Bi-Directional Causality.

(3).**No Directional Causality** :

-> This causality is not a significant above 2 directional so is called no directional causality.

#### **The Granger Causality Test :**

Let Y(t) be the time series for which the future values have to be predicted and let X(t) be the other time series that will be used to augment Y(t) with the lagged values of X(t). The Granger Causality Test is based on univariate and multivariate AR models which assume the time series are stationary.  Stationary can be confirmed by applying the augmented Dickey-Fuller test. If the series are not stationary, they should be transformed into stationary series through operations such as detrending, differencing, or log transforming prior to applying the test.  The GCT uses a series of t-tests and F-tests conducted on the lagged values of Y(t) and X(t). The test is based on the following null hypothesis.

The test is based on the following OLS regression model:



Here, the αj and βj are the regression coefficients and εi is the error term. The test is based on the null hypothesis:

H0: β1 = β2 = … = βm = 0

We say that x Granger-causes y when the null hypothesis is rejected.

We use the usual F test described in Adding Extra Variables to a Regression Model to determine whether there is a significant difference between the regression model shown above (the full model) or the reduced model, based on the null hypothesis, without the βj terms (i.e. where all the βj = 0).

There we demonstrate two equivalent forms of the test:



Here, all the terms are based on the full model with the exception of SS′E and Rr2, which are based on the reduced model.

If the p-value for this test is less than the designed value of α, then we reject the null hypothesis and conclude that x causes y (at least in the Granger causality sense).

**Assumptions**

* The Granger Causality test assumes that both the x and y time series are stationary. If this is not the case, then differencing, de-trending or other techniques must first be employed before using the Granger Causality test.
* Note that the number of lags, i.e. the value of m, is critical, in that different values of m may lead to different test results. One approach to selecting an appropriate value for m is to choose the value that results in the full model with the smallest AIC or BSC value.
* It is possible that causation is only in one direction, or in both directions (x Granger-causes y and y Granger causes x) or in neither direction.

## Spurious Regression

Linear regression might indicate a strong relationship between two or more variables, but these variables may be totally unrelated in reality. Predictions fail when it comes to domain knowledge, this scenario is known as spurious regression.

There is a strong relationship between chicken consumption and crude oil exports in the below graph even though they are unrelated.





* Consider the above time-series graph, variable X has a direct influence on variable Y but there is a lag of 5 between X and Y in which case we cant use the correlation matrix. For eg. an increase in coronavirus positive cases in the city and an increase in the number of people getting hospitalized. For better forecasting, here we would like to know if there is a causal relationship.

## *Granger Causality comes to Rescue*

Prof. Clive W.J. Granger, recipient of the 2003 Nobel Prize in Economics developed the concept of causality to improve the performance of forecasting.

It is basically an econometric hypothetical test for verifying the usage of one variable in forecasting another in multivariate time series data with a particular lag.

A prerequisite for performing the Granger Causality test is that the data need to be stationary i.e it should have a constant mean, constant variance, and no seasonal component. Transform the non-stationary data to stationary data by differencing it, either first-order or second-order differencing. Do not proceed with the Granger causality test if the data is not stationary after second-order differencing.

Let us consider three variables *Xt* , *Yt* , and *Wtpreset in time series data.*

Case 1: Forecast  Xt+1 based on past values Xt.

Case 2: Forecast Xt+1 based on past values Xt and Yt.

Case3 : Forecast Xt+1 based on past values Xt , Yt , and Wt, where variable Ythas direct dependency on variable Wt.

Here Case 1 is univariate time series also known as the autoregressive model in which there is a single variable and forecasting is done based on the same variable lagged by say order p.

**The equation for the Auto-regressive model of order p (RESTRICTED MODEL, RM)**

Xt = α + 𝛾1 X𝑡−1 + 𝛾2X𝑡−2 + ⋯ + 𝛾𝑝X𝑡−𝑝

where  p parameters (degrees of freedom) to be estimated.

In Case 2 the past values of Y contain information for forecasting Xt+1. Yt is said to “Granger cause” Xt+1 provided Yt occurs before Xt+1and it contains data for forecasting Xt+1.

**Equation using a predictor Yt (UNRESTRICTED MODEL, UM)**

Xt = α + 𝛾1 X𝑡−1 + 𝛾2X𝑡−2 + ⋯ + 𝛾𝑝X𝑡−𝑝+ α1Yt-1+  ⋯ + α𝑝Yt-p

2p parameters (degrees of freedom) to be estimated.

If Y*t* causes X*t*, then Y must precede X which implies:

* Lagged values of Y should be significantly related to X.
* Lagged values of X should not be significantly related to Y.

*Case 3 can not be used to find Granger causality since variable Yt is influenced by variable Wt.*

## Hypothesis test

Null Hypothesis (H0) : *Yt* does not “Granger cause” *Xt*+1 i.e., 𝛼1 = 𝛼2 = ⋯ = 𝛼𝑝 = 0

Alternate Hypothesis(HA): *Yt* does “Granger cause” *Xt*+1, i.e., at least one of the lags of Y is significant.

### **Calculate the f-statistic**

Fp,n-2𝑝−1 = (𝐸𝑠𝑡𝑖𝑚𝑎𝑡𝑒 𝑜𝑓 𝐸𝑥𝑝𝑙𝑎𝑖𝑛𝑒𝑑 𝑉𝑎𝑟𝑖𝑎𝑛𝑐𝑒) / (𝐸𝑠𝑡𝑖𝑚𝑎𝑡𝑒 𝑜𝑓 𝑈𝑛𝑒𝑥𝑝𝑙𝑎𝑖𝑛𝑒𝑑 𝑉𝑎𝑟𝑖𝑎𝑛𝑐𝑒)

 Fp,n-2𝑝−1 =  ( (𝑆𝑆𝐸𝑅𝑀−𝑆𝑆𝐸𝑈𝑀) /𝑝) /(𝑆𝑆𝐸𝑈𝑀 /𝑛−2𝑝−1)

*where n is the number of observations and  
SSE is Sum of Squared Errors.*

If the p-values are less than a significance level (0.05) for at least one of the lags then reject the null hypothesis.

Perform test for both the direction Xt->Yt and Yt->Xt.

Try different lags (p). The optimal lag can be determined using AI

## Limitation

* Granger causality does not provide any insight on the relationship between the variable hence it is not true causality unlike ’cause and effect’ analysis.
* Granger causality fails to forecast when there is an interdependency between two or more variables (as stated in Case 3).
* Granger causality test can’t be performed on non-stationary data.

### **Resolving Chicken and Egg problem**

## Let us apply Granger causality to check whether the egg came first or chicken came first.

### Importing libraries

### **import** matplotlib.pyplot **as** plt

**import** seaborn **as** sns

**import** numpy **as** np

**import** pandas **as** pd

## Loading Dataset

df **=** pd**.**read\_excel('C:\\Users\\Vandit\\Desktop\\data.xlsx')

df

|  | **Year** | **chicken** | **egg** |
| --- | --- | --- | --- |
| **0** | 1930 | 468491 | 3581 |
| **1** | 1931 | 449743 | 3532 |
| **2** | 1932 | 436815 | 3327 |
| **3** | 1933 | 444523 | 3255 |
| **4** | 1934 | 433937 | 3156 |
| **5** | 1935 | 389958 | 3081 |
| **6** | 1936 | 403446 | 3166 |
| **7** | 1937 | 423921 | 3443 |
| **8** | 1938 | 389624 | 3424 |
| **9** | 1939 | 418591 | 3561 |
| **10** | 1940 | 438288 | 3640 |
| **11** | 1941 | 422841 | 3840 |
| **12** | 1942 | 476935 | 4456 |
| **13** | 1943 | 542047 | 5000 |
| **14** | 1944 | 582197 | 5366 |
| **15** | 1945 | 516497 | 5154 |
| **16** | 1946 | 523227 | 5130 |
| **17** | 1947 | 467217 | 5077 |
| **18** | 1948 | 499644 | 5032 |
| **19** | 1949 | 430876 | 5148 |
| **20** | 1950 | 456549 | 5404 |
| **21** | 1951 | 430988 | 5322 |
| **22** | 1952 | 426555 | 5323 |
| **23** | 1953 | 398156 | 5307 |
| **24** | 1954 | 396776 | 5402 |
| **25** | 1955 | 390708 | 5407 |
| **26** | 1956 | 383690 | 5500 |
| **27** | 1957 | 391363 | 5442 |
| **28** | 1958 | 374281 | 5442 |
| **29** | 1959 | 387002 | 5542 |
| **30** | 1960 | 369484 | 5339 |
| **31** | 1961 | 366082 | 5358 |
| **32** | 1962 | 377392 | 5403 |
| **33** | 1963 | 375575 | 5345 |
| **34** | 1964 | 382262 | 5435 |
| **35** | 1965 | 394118 | 5474 |
| **36** | 1966 | 393019 | 5540 |
| **37** | 1967 | 428746 | 5836 |
| **38** | 1968 | 425158 | 5777 |
| **39** | 1969 | 422096 | 5629 |
| **40** | 1970 | 433280 | 5704 |

df**.**head() *## return first 5 rows in df*

|  | **Year** | **chicken** | **egg** |
| --- | --- | --- | --- |
| **1** | 1931 | 449743 | 3532 |
| **2** | 1932 | 436815 | 3327 |
| **3** | 1933 | 444523 | 3255 |
| **4** | 1934 | 433937 | 3156 |
| **5** | 1935 | 389958 | 3081 |

df**.**dtypes

Year int64

chicken int64

egg int64

dtype: object

df**.**describe()

|  | **Year** | **chicken** | **egg** |
| --- | --- | --- | --- |
| **count** | 41.000000 | 41.000000 | 41.000000 |
| **mean** | 1950.000000 | 428343.853659 | 4787.804878 |
| **std** | 11.979149 | 49514.639114 | 927.864894 |
| **min** | 1930.000000 | 366082.000000 | 3081.000000 |
| **25%** | 1940.000000 | 390708.000000 | 3640.000000 |
| **50%** | 1950.000000 | 423921.000000 | 5322.000000 |
|  |  |  |  |
| **75%** | 1960.000000 | 444523.000000 | 5435.000000 |
| **max** | 1970.000000 | 582197.000000 | 5836.000000 |

#Draw Plot

def plot\_df(df, x, y, title="", xlabel='Date', ylabel='Value', dpi=100):

plt.figure(figsize=(10,3), dpi=dpi)

plt.plot(x, y, color='tab:red')

plt.gca().set(title=title, xlabel=xlabel, ylabel=ylabel)

plt.show()

plot\_df(df, x=df.Year, y=df.chicken, title='Polulation of the chicken across US')

plot\_df(df,x=df.Year, y=df.egg, title='Egg Produciton')

**Augmented Dickey-Fuller Test (ADF test)**

**Null Hypothesis (H0): Series has a unit root and is non-stationary.**

**Alternative Hypothesis (HA): Series has no unit root and is stationary.**

from statsmodels.tsa.stattools import adfuller

result=adfuller(df['chicken'])

print(f'Test statistics:{result[0]}')

print(f'p-value:{result[1]}')

print(f'critical\_values:{result[4]}')

Test statistics:-1.8986588103410362

p-value:0.33269231149362943

critical\_values:{'1%': -3.6155091011809297, '5%': -2.941262357486514, '10%': -2.6091995013850418}

if result1[1] > 0.05: print("Series is not stationary")

else: print("Series is stationary")

Series is not stationary

result = adfuller(df['egg'])

print(f'Test Statistics: {result[0]}')

print(f'p-value: {result[1]}')

print(f'critical\_values: {result[4]}')

Test Statistics: -1.2700853266987826

p-value: 0.6427448032671643

critical\_values: {'1%': -3.610399601308181, '5%': -2.939108945868946, '10%': -2.6080629651545038}

if result[1]>0.05: print("Series is not stationary")

else:print("Series is stationary")

Series is not stationary

## Data Transformation

df\_transformed = df.diff().dropna()

df=df.iloc[1:]

print(df.shape)

df\_transformed.shape

(40, 3)

Out[46]:

(40, 3)

df\_transformed.head()

|  | **Year** | **chicken** | **egg** |
| --- | --- | --- | --- |
| **1** | 1.0 | -18748.0 | -49.0 |
| **2** | 1.0 | -12928.0 | -205.0 |
| **3** | 1.0 | 7708.0 | -72.0 |
| **4** | 1.0 | -10586.0 | -99.0 |
| **5** | 1.0 | -43979.0 | -75.0 |

plot\_df(df\_transformed,x=df.Year,y=df\_transformed.chicken,title='population of the chicken across in US')

plot\_df(df\_transformed, x=df.Year, y=df\_transformed.egg, title='Egg Produciton')

## Repeat the ADF test again on differenced data to check for stationarity.

result = adfuller(df\_transformed['chicken'])

print(f'Test Statistics: {result[0]}')

print(f'p-value:{result[1]}')

print(f'critical\_values: {result[4]}')

if result[1] > 0.05: print("Series is not stationary")

else: print("Series is stationary")

Test Statistics: -3.782497006630482

p-value:0.003091556771872362

critical\_values: {'1%': -3.6155091011809297, '5%': -2.941262357486514, '10%': -2.6091995013850418}

Series is stationary

result = adfuller(df\_transformed['egg'])

print(f'Test Statistics: {result[0]}')

print(f'p-value:{result[1]}')

print(f'critical\_values: {result[4]}')

if result[1] > 0.05: print("Series is not stationary")

else: print("Series is stationary")

Test Statistics: -3.9304148180643623

p-value:0.0018220700886365682

critical\_values: {'1%': -3.610399601308181, '5%': -2.939108945868946, '10%': -2.6080629651545038}

Series is stationary

## Transformed chicken and egg data are stationary, hence there is no need to go for second-order differencing.

## Test the Granger Causality

### There are several ways to find the optimal lag but for simplicity let’s consider 4th lag as of now.

# Do eggs granger cause chickens?

## Null Hypothesis (H0) : eggs do not granger cause chicken.

## Alternative Hypothesis (HA ) : eggs granger cause chicken.

from statsmodels.tsa.stattools import grangercausalitytests

grangercausalitytests(df\_transformed[['chicken', 'egg']], maxlag=4)

Granger Causality

number of lags (no zero) 1

ssr based F test: F=10.9970 , p=0.0021 , df\_denom=36, df\_num=1

ssr based chi2 test: chi2=11.9134 , p=0.0006 , df=1

likelihood ratio test: chi2=10.3960 , p=0.0013 , df=1

parameter F test: F=10.9970 , p=0.0021 , df\_denom=36, df\_num=1

Granger Causality

number of lags (no zero) 2

ssr based F test: F=4.4342 , p=0.0197 , df\_denom=33, df\_num=2

ssr based chi2 test: chi2=10.2121 , p=0.0061 , df=2

likelihood ratio test: chi2=9.0449 , p=0.0109 , df=2

parameter F test: F=4.4342 , p=0.0197 , df\_denom=33, df\_num=2

Granger Causality

number of lags (no zero) 3

ssr based F test: F=3.5308 , p=0.0265 , df\_denom=30, df\_num=3

ssr based chi2 test: chi2=13.0639 , p=0.0045 , df=3

likelihood ratio test: chi2=11.1881 , p=0.0108 , df=3

parameter F test: F=3.5308 , p=0.0265 , df\_denom=30, df\_num=3

Granger Causality

number of lags (no zero) 4

ssr based F test: F=4.7569 , p=0.0049 , df\_denom=27, df\_num=4

ssr based chi2 test: chi2=25.3701 , p=0.0000 , df=4

likelihood ratio test: chi2=19.2025 , p=0.0007 , df=4

parameter F test: F=4.7569 , p=0.0049 , df\_denom=27, df\_num=4

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[0., 0., 0., 0., 0., 0., 1., 0., 0.],

[0., 0., 0., 0., 0., 0., 0., 1., 0.]])])}

## Conclusion--p value is low so,null hypothesis is rejected so,egg granger causing chicken

### That implies Eggs Came First

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Thank You